Turbulent mixing at the top of stratocumulus clouds

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The process of mixing at the top of a turbulent cloud layer contains a new feature which has not been considered in previous studies of mixing; evaporation of liquid water can cause density changes which may affect the dynamics. A model experiment has been devised to study this problem, using liquids whose density behaviour is non-linear to simulate evaporation.

The existence of a moist, stable, turbulent region above cloudtop can be explained using the laboratory results, which suggest that this region can be regarded dynamically as part of the cloud. Comparison of the rates of mixing in the model experiments with and without 'evaporation' suggests that evaporation could cause a small decrease in the mixing rate for a given density difference, but the change would be negligible in practice. This result also sheds some light on the mechanism of mixing, in both the linear and non-linear cases.

1. Introduction

The problem of mixing across a stable interface between two fluids, one of which is turbulent and the other at rest, is of importance in many geophysical contexts. The top layers of the ocean and lakes are set into turbulent motion by wind or other effects near the surface, and mix downwards into the deeper layers; the existence of a sharp thermocline can be explained in this way (Proudman 1953). It has also been suggested (Ball 1960) that the changes in the height of atmospheric inversions can be attributed to the mixing action of convective turbulence below the inversion, which continually incorporates air from above it into the convecting region.

Another situation which is very similar to this latter case, but which seems to have received less attention, is found at the top of a layer of stratocumulus cloud. The conditions under which such a cloud is formed need not be discussed in detail here, but we suppose that a general lifting of moist air over a large area has resulted in the formation of an extensive layer of cloud, above which there is drier air. Convective turbulence in the layer must produce mixing between the cloud and the drier air above it, and this in the absence of subsidence would cause the cloud tops to rise. The object of this paper is to examine the cloudtop mixing using model experiments relevant to this case, and to try to explain the observed

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structure of the air above stratocumulus clouds. James (1959), for example, has reported that the first few hundred feet above a stratocumulus cloud layer is typically a region of temperature inversion, in which the humidity falls off gradually between saturation and the environmental values. In this layer the air is turbulent, with the amplitude of fluctuations falling gradually from that characteristic of cloud to zero in the air upwards of 500 ft. above the visible cloud top. It is not at all obvious how the motion in the cloud could affect the clear air above it in this way.

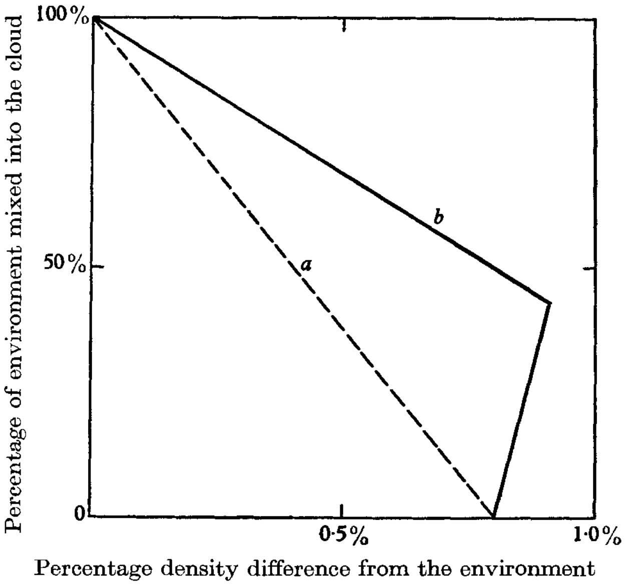


FIGURE 1. The density behaviour of cloudy air containing 1 g/kg liquid water, mixing with an environment 2 °C warmer which (*a*) is saturated, (*b*) has a relative humidity of 50 %. Cloud at 0° C and 700 mb.

We should first consider how this mixing problem might be expected to differ from those previously treated or, in other words, why laboratory results obtained with other problems in mind cannot be applied immediately to clouds. The reason is that a new physical effect is introduced when cloudy and unsaturated air are mixed. The evaporation of a cloud into relatively dry air near its top can cause cooling and consequently a density change; for example, the density behaviour is shown in figure 1 for cloudy air with a concentration of liquid water of 1 g/kg mixing in various proportions with air of 50 % relative humidity which is initially 2 °C warmer than the cloud. This is highly non-linear, with the maximum density occurring just at the point where the air is saturated but all the liquid water has evaporated, i.e. at the visible edge of the cloud. Mixing therefore can in this case produce fluid which is not intermediate in properties between the two components, but has a density greater than either, and we cannot say without experiment whether this will affect the rate of mixing.

This density increase can certainly affect the motion round cumulus clouds; evaporative cooling produces a thin layer of dense air which can often be observed as a pronounced downdraught at the edge of a tower growing into dry air aloft. Indeed Squires (1958) has gone further and proposed a mechanism whereby evaporation could control the rate of mixing deep inside a cumulus cloud. Eventually we wish to gain some understanding of this process too, but as a first step it seems desirable to consider the simpler situation found above a strato-cumulus layer, which is more nearly related to the cases previously mentioned.

2. Comparison with previous experiments

The basic laboratory experiments for the case of *linear* density changes were conducted by Rouse & Dodu (1955). By this we mean that the fluids they used, solutions of common salt and sugar in water, were such that the density of intermediate mixtures changed linearly between the densities of the two components, i.e. there were no volume changes on mixing. They investigated systematically the dependence of the rate of mixing on Reynolds number and an internal Froude number. (This latter is equivalent to the inverse square root of a Richardson number, a stability parameter which we shall define and use in a later section.) Two fluids of different density were placed in a cylindrical tank, with a stable interface between them. The fluid on one side of the interface was stirred mechanically, and mixing was observed to take place by the incorporation of still fluid into the turbulent region by the breaking of waves at the interface. The mixing rate was measured by recording both the amount of fluid which had to be removed to keep the interface at a constant level, and also the concentration of the unstirred fluid in the mixture.

The present paper can be regarded as an extension of this investigation, using very similar apparatus and methods which we shall describe more fully later. The new feature which must be introduced is the equivalent of the process of evaporation in the atmosphere. That is, we must now work with liquids whose density changes in a non-linear manner on mixing, because of a decrease in volume. We shall for conciseness refer to the two types of experiment as 'linear' and 'non-linear' respectively. No attempt will be made here to cover the wider range of conditions used by Rouse & Dodu, and reference should be made to their discussion of the non-dimensional presentation of results. Instead we shall keep fixed all parameters except the density difference and its degree of non-linearity, and investigate the importance of the non-linearity by direct comparison with the corresponding linear experiments. The model experiments allow us to explain several features observed in the atmospheric case. The results also shed considerable light on the mechanism of mixing in both the linear and non-linear conditions.

3. The production of non-linear density changes

Mixtures of alcohol and water have a highly non-linear density behaviour, and this has in fact led in the past to inaccuracies in convection experiments where alcohol has been used as a source of buoyancy in a tank of water. We can now make use of this non-linearity to produce a laboratory model of evaporation. Alcohol and water have much too high a density difference to be used directly, so we sought combinations of these two fluids with other substances which would give smaller density differences and similar viscosities, while retaining the desired property.

The combinations finally found most suitable were as follows. For the lower layer, which corresponds to the cloud and was the one stirred in all the experiments reported here, we used a mixture of 80 % water and 20 % methylated spirits by volume, plus a small quantity of glycerine to increase both the viscosity and density slightly. The top unstirred layer consisted of a mixture of 20 % water and 80 % methylated spirits with potassium iodide dissolved in sufficient quantity to bring the density difference to the desired value. This salt was chosen because it is very soluble in both alcohol and water; it enabled the desired range of density differences to be covered with a nearly constant bottom mixture and similar non-linear properties in each case. An incidental advantage was that the concentration of salt in the stirred fluid could be used as a measure of the mixing.

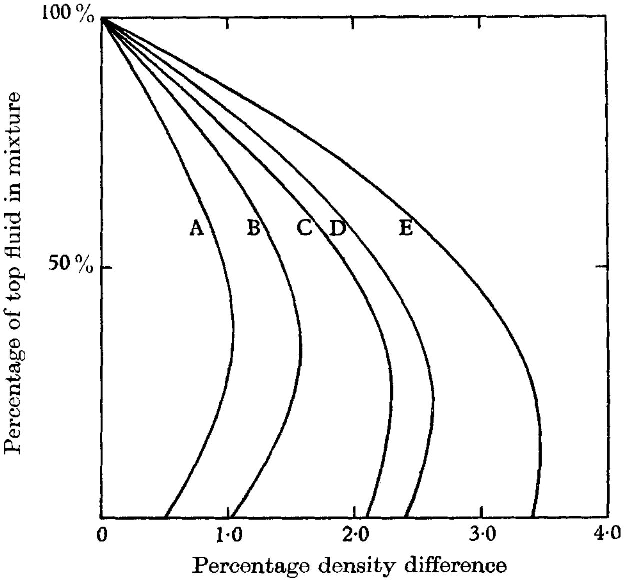


FIGURE 2. The measured density behaviour of mixtures of the 'non-linear' fluids used in our experiments. The curves have been plotted for clarity with respect to the density of the top layer, although in practice both densities were varied by a few percent to obtain the desired behaviour.

The density behaviour of mixtures of the pairs of fluids which were actually used in five of our experiments is shown in figure 2. These reproduce the magnitude of the non-linear density changes in clouds quite well, but they differ from the behaviour shown in figure 1 for mixtures of cloudy and unsaturated air by being more nearly parabolic than triangular in shape.

These initial mixtures also resulted in the viscosities of the two fluids being very closely equal, just under three times that of water. Using a standard flow method for measuring kinematic viscosities, the time of flow of a standard volume was kept in the range 210–220 sec, compared with the time for water at 20 °C of about 89 sec. A point to be kept in mind in the interpretation of our results is that the viscosity as well as density of our mixtures was a non-linear function of mixing. The extreme changes of viscosity of intermediate mixtures amounted to about 20 %, as shown in figure 3. This is an unwanted effect which complicates the experiment, but we believe that the effects due to these viscosity changes can be distinguished from those due to the non-linearity of density which we wish to investigate.

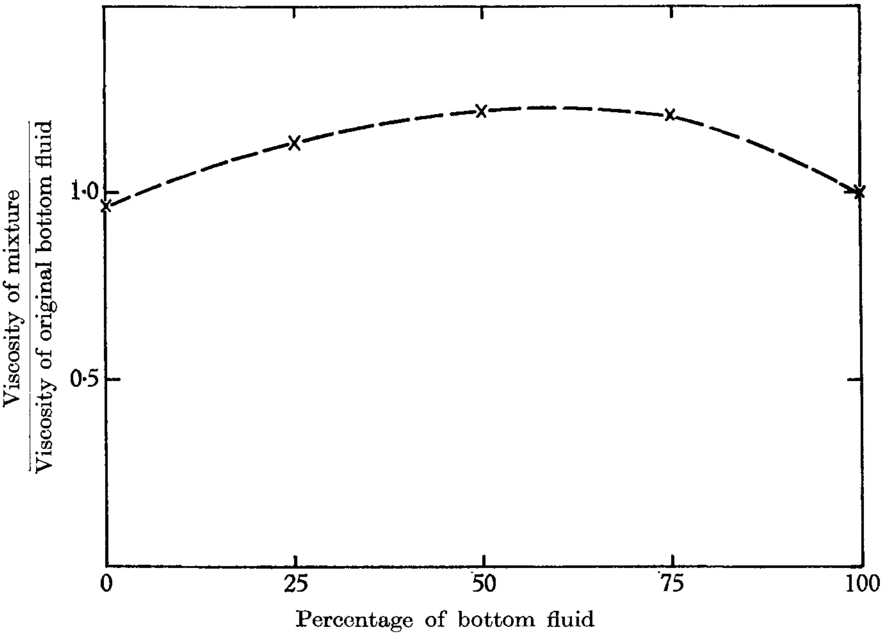


FIGURE 3. Typical variation of viscosity with the proportions of the mixtures. The maximum variation from the standard value was about 20 %.

For the purpose of direct comparison under the same experimental conditions, linear-mixing experiments have also been performed. The fluids used were prepared using water and alcohol in nearly the same proportions for each layer, with glycerine and common salt to adjust the viscosity and density and provide a means of measuring the mixing rate. The (kinematic) viscosities of both layers were kept the same, and the same as that used in the non-linear experiments. In this case the viscosity remained very nearly constant during mixing. The absolute densities, as well as the density differences, were also kept within a few percent of the same values in the two series of experiments.

4. The experimental method

The experiments were carried out in a Perspex cylinder 20 cm in diameter and 40 cm deep. Provision was made for filling and draining, and inserting the stirring device, which consisted of an array of flat metal strips 1 cm wide at 5 cm spacing, similar to that used by Rouse & Dodu. This was oscillated through a distance of 1 cm perpendicular to its plane at the fixed rate of 5 c/s throughout the experiment. The bottom half of the tank was stirred in all cases, with the grid 9·5 cm from the floor and 9 cm from the interface. This was kept at the same level as mixing proceeded by removing fluid from the bottom of the tank, so that the intensity of turbulence (which must depend on the distance from the stirrer) would remain constant at the level of the interface.

So that the tank could be filled conveniently, a thin removable Perspex partition was provided across the centre. Without this, the non-linear density changes can cause considerable mixing even before the mechanical stirring is begun. The top half of the tank was not kept refilled and the level of fluid in it fell during the experiment, but care was taken always to have sufficient fluid above the interface so that the mixing rate depended on the geometry of only the lower fluid, and the depth of the upper layer could be regarded as unlimited.

The amount of mixing was measured by withdrawing small samples of the mixture from the level of the stirrer, and measuring the concentration of the upper fluid in it using a conductivity cell. From the concentrations at successive intervals of time we can deduce the rate of mixing, and this has been expressed as an instantaneous mean velocity of rise of the interface or of diffusion of stationary fluid into the stirred region. The conductivity cell was calibrated directly for each experiment using known dilutions of the upper and lower fluids; at the same time, the density of mixtures covering the whole range of dilutions was determined using a hydrometer to give the plots already shown in figure 2. During several runs we also measured the velocity directly by recording the level of fluid in the tank each time the interface was adjusted, and the density and viscosity of the bottom fluid was also measured.

5. Experimental observations

We shall first describe our visual observations of the mixing process in the linear and non-linear cases. In order to follow the behaviour of the interface, dye was added to the bottom (stirred) fluid, and the 'visual interface', which was kept at a fixed level by removing mixed fluid, represented the extreme height to which dyed fluid had advanced.

The most striking discovery of Rouse & Dodu was that the fluid on the unstirred side of the interface during 'linear' mixing remains nearly at rest, with little agitation except very near the interface. Mixing takes place through the breaking of waves from the stirred region into the unstirred. When mechanical agitation is stopped, some of the streamers thrown upwards fall back to form a thin transition region, but otherwise the interface remains sharp. We have repeated and agree with these observations, but in addition would suggest that there is a transition region *below* the visual interface which can be detected by withdrawing samples from various levels; the density gradient in the top centimetre of the dyed layer was in our experiments much higher than it was near the stirrer.

When the ordinary fluids are replaced by a pair which are statically stable when unmixed, but for which a mixture in some proportion is heavier than either component (see figure 2), several important differences appear. First, mixing becomes possible with *no* mechanical agitation of the lower fluid at all. A small amount of molecular diffusion produces streamers of heavy fluid which fall through the lower layer, and thus produce further agitation and mixing. Though this is an interesting phenomenon in itself (and one which is perhaps important in nature) it is difficult to model properly since it is so sensitive to molecular effects. All our quantitative experiments relate to the case where agitation from some source other than the interfacial mixing itself is dominant; this would correspond to the conditions at the top of a cloud which is in vigorous convective motion driven from below, and not solely by evaporation at its top.

With mechanical agitation the interface between non-linear fluids looks much the same as it does in the linear case; figure 4, plate 1, is an illustrative photograph taken during a non-linear experiment. It was observed visually (though difficult to show photographically) that the interface appeared rather smoother during the non-linear experiments. This effect we attribute to the formation of a region of more stable density gradient just at the top of the stirred layer. The existence of such a stable layer may be inferred from our observation of an effect which does not occur at all in the linear case. When mixing is first begun with non-linear fluids, there appears above the original interface a lighter but definite band of dye, about 1 cm deep, with sharp boundaries between the clear fluid above and the darker dye below. This gradually becomes more intense, and finally merges into the lower layer; its top then becomes the interface followed visually.

The suggested mechanism of formation of this intermediate layer is related to the spatial differences of concentration which are implied by non-linear density profiles such as those shown in figures 1 and 2. When turbulent elements from the stirred layer are thrown upwards a long way, so that the resulting mixture with the upper layer contains a large proportion of upper fluid, a region will be produced which is more stable than in the corresponding linear case, but otherwise similar to it. On the other hand, for small amounts of upper fluid mixed into the lower and therefore at levels closer to the lower fluid, the density gradient produced by mixing will be unstable, and a rapid mixing with the stirred fluid will take place. Because of this abrupt change between stable and unstable stratification at a certain concentration, there will be in the non-linear case a second sharp interface at the bottom of the transition region, and this is what we observe. We shall have more to say about this phenomenon in § 7; its relation to the cloud observations is shown diagrammatically in figure 8.

6. Quantitative results

A major aim of the present experiments has been to compare the rate of mixing between fluids with and without non-linear density changes. To do this we have standardized our conditions, keeping the same geometry, agitation rate and viscosity of the two fluids, while changing only the density difference and its degree of non-linearity.

The quantity measured directly was the concentration of the upper fluid in the lower as a function of time, and from this the density difference and the mixing velocity may easily be obtained. For the purposes of comparison with the non-linear experiments we have plotted in figure 5 the density difference as a function of time for three linear cases; the results are seen to be similar to those given by Rouse & Dodu, with the density difference decreasing with time. Also in figure 5 are the corresponding experimental results for various non-linear cases, plotted using figure 2. These latter cover the same range of initial density differences, but the behaviour as a function of time is quite different; the density may increase, decrease or stay nearly constant, as is indeed obvious from figure 2.

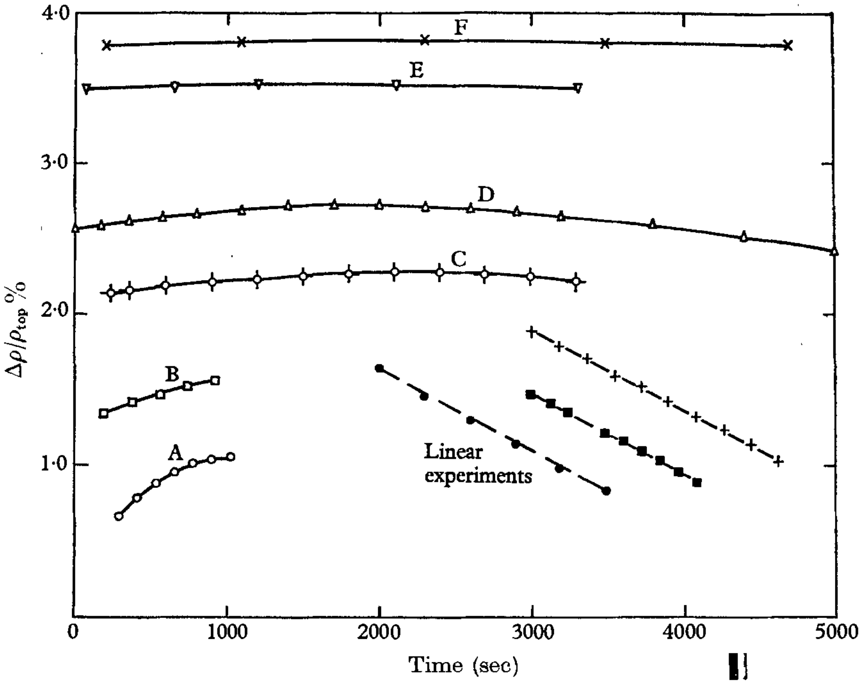


FIGURE 5. The percentage changes in density difference as a function of time in our laboratory experiments. The origin of time is arbitrary, but the times correspond in figures 5 and 6. Dashed lines are used for linear experiments and full curves for non-linear experiments. The letters on the curves refer to figure 2, where the corresponding densities are plotted. Case F (not shown in figure 2) was similar to E.

The velocity of mixing V may be obtained to sufficient accuracy from the equation of continuity, ignoring volume changes, which gives

$$V = d(b-a)/t(1-b), \tag{1}$$

where d is the depth of the lower fluid, and a, b are the concentrations of the upper fluid in the mixture, at the beginning and end of an interval of time t. In figure 6 these velocities are plotted against time for the same linear and non-linear cases as in figure 5. The linear experiments all show an increasing velocity whereas the non-linear ones can have a velocity either increasing or decreasing with time.

The most informative plot of these results is shown in figure 7, where the instantaneous mixing velocities are plotted against density difference using figures 5 and 6, and also the results of two more linear experiments at higher density differences which were omitted from the earlier figures for the sake of clarity. All the 'linear' experiments lead to results lying satisfactorily along a single curve; the differences between runs give an indication of the over-all accuracy attained. They are in fact equivalent to one of Rouse & Dodu's experiments, in which the mixing was followed for a longer time. Note that over the range of density differences covered here the non-linear results all lie below the linear curve. Thus the changes in mixing velocity cannot be explained merely in terms of density differences as it can for linear experiments; there is a real decrease in the mixing rate which must be explained by the non-linear properties of our fluids.

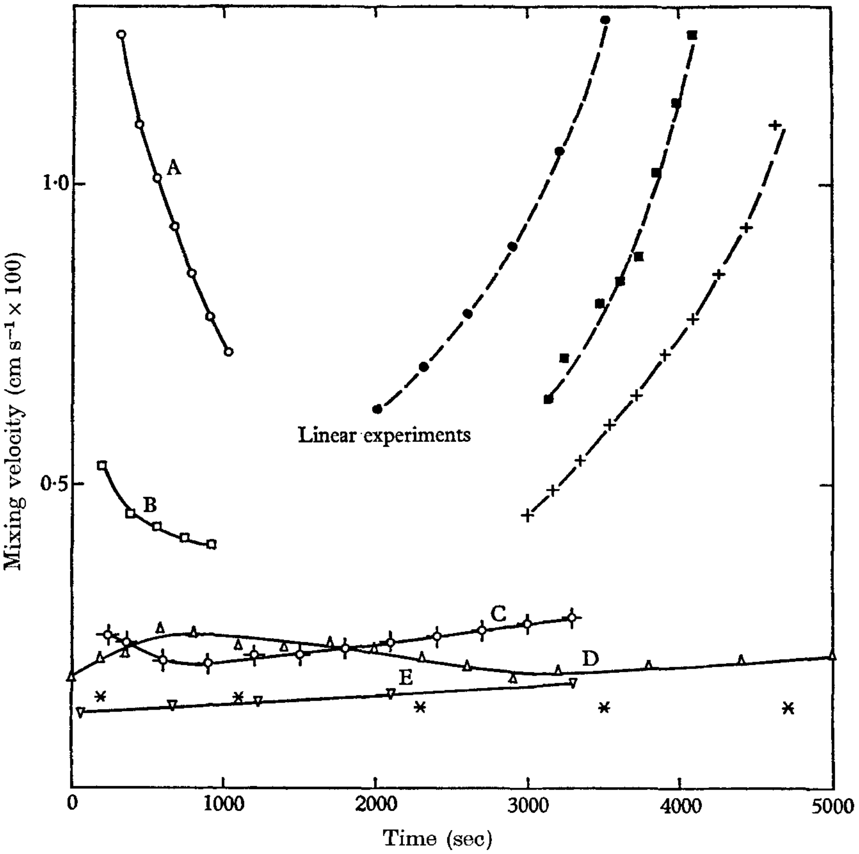


FIGURE 6. Mixing rates as a function of time. The symbols refer to the same runs as in figure 5. Dashed lines: linear experiments; full lines: non-linear experiments.

There are three factors which can cause the change in behaviour compared with the linear case. First, and most obviously, a decrease in mixing rate with time is caused by an increase in density difference produced by the mixing (see, for example, case A in figures 5–7). If this were the only effect, the non-linear results in figure 7 would lie along the 'linear' curve, though the points would be traversed in a different time sequence.

Secondly, there is the main effect which we wish to examine, the influence of the magnitude of the non-linearity of density. This can be defined as, say, the maximum difference in density of any non-linear mixture from the corresponding linear density, found by linear interpolation of the original densities of the upper and lower fluids. This quantity was about the same at the beginning of all our non-linear experiments, but it clearly decreases with time, and its influence might be expected to decrease correspondingly. We believe that this change in non-linearity of density is the explanation of the results of Expt. E, in which the density difference remained substantially constant, while the velocity increased with time towards the linear value. The behaviour of Expt. C is explained by a combination of these two effects, with the velocity first decreasing as the density difference increased, and then increasing again as the non-linearity decreased.

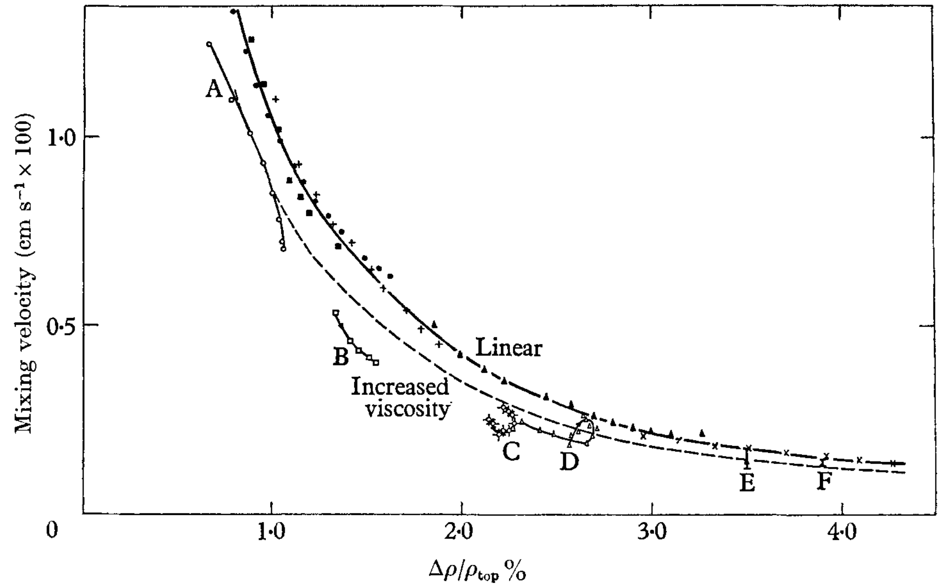


FIGURE 7. Mixing rates plotted as a function of density difference for both linear and non-linear runs. The symbols refer to the same cases as in figures 5 and 6. The dashed curve shows the decreased rate to be expected for a 20 % increase in viscosity.

There is a third effect which is not present in the cloud top mixing which we wish to investigate, and which tends partly to obscure the two caused by density differences in our model experiments. The viscosity changes with composition of the mixture, as shown in figure 3, and this too can have an effect on the mixing rate. The viscosity of the mixture was not recorded continuously in all experiments, but by using the measured maximum change of 20 % in viscosity we can put an upper limit on the changes in rate which could be due to viscosity alone. This we have done using Rouse & Dodu's results to find the functional form of the dependence of mixing velocity on viscosity, and the resulting decrease in rate from the linear value is shown as a dotted line below the full curve.

We believe that this allowance for viscosity changes is in fact an overestimate, since its use is equivalent to assuming that the viscosity is immediately altered everywhere in the stirred fluid by this amount. Actually the viscosity is higher only in a thin layer near the interface, and gradually increases in the body of the stirred fluid. This impression is confirmed by the results of Expt. D which was continued for a longer time than other runs, and in which the viscosity was recorded at frequent intervals. The mixing rate at the beginning of the experiment was lower than the linear value. As mixing reduced the non-linearity of density (without at first increasing the viscosity of the lower layer significantly) the mixing rate increased in spite of a small increase in density difference. As the viscosity of the mixture increased, the mixing rate again fell to values below the linear values and finally, with decreasing density, the mixing rates followed a curve similar to the linear one for an increased viscosity. The maximum viscosity in this case corresponded to a viscometer reading of 266 sec.

Taking all these factors into account, we can therefore conclude from figure 7 that most of the decrease of mixing rate in the non-linear experiments, compared with the linear rate at the same density difference, can only be explained in terms of the non-linear changes in density.

7. Discussion

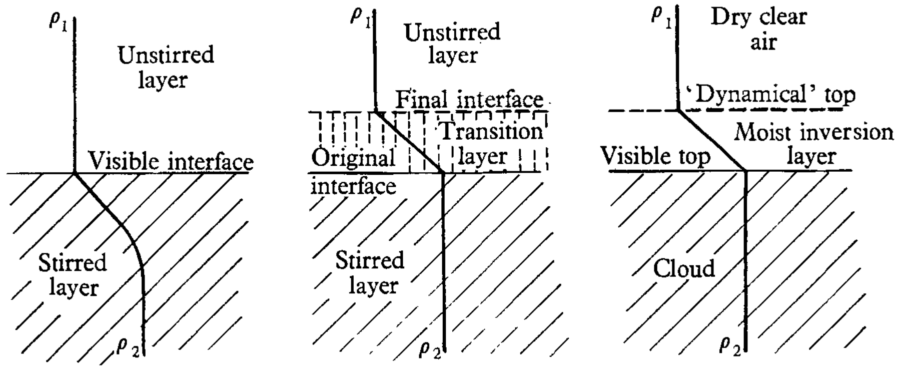
Finally we shall comment on the results of the last two sections, and discuss their relevance to the mixing at the top of the stratocumulus cloud, which has provided the impetus for this study.

Our qualitative observations of the nature of the interface, and in particular the development of the stable transition region above the interface between 'non-linear' liquids, seem to provide a satisfactory explanation of the measurements made by James (1959), and reported in § 1. Let us suppose that initially we have a sharp interface between air containing condensed water and unsaturated air above it. If this is identified with the original interface between the bottom (dyed) and top (colourless) fluids, then the laboratory results suggest that as mixing and evaporation take place, a stable, moist region will be formed above this interface. In the laboratory case this region is made visible because dye is carried with the bottom fluid as it mixes, whereas water vapour derived from the cloud becomes invisible unless the air is saturated. The calculations leading to figure 1 show that a stable region formed in this way will begin at the visible edge of the cloud, which is also the density maximum. This cloud-top inversion will be maintained by the convective stirring in the cloud layer, in the way we have discussed at the end of § 5. The relation between the three cases considered is shown in figure 8.

We do not of course suggest that processes such as subsidence or radiation cooling of the top of the cloud sheet are unimportant, but our experiments have shown how all the effects observed by James can be explained, qualitatively at least, in terms of the dynamics of the cloud as it mixes upwards. The stable, moist region above cloud-top can properly be regarded as dynamically part of the cloud, and the existence of turbulence in it is therefore less surprising.

The quantitative results of § 6 also show that the rate of mixing of a cloud layer into the air above it can be expected to be slightly less than if no evaporation were taking place. The differences will in fact be negligible in practice, especially since so little is known about the structure of turbulence in clouds.

Although we have been concerned in this paper mainly with the effects of evaporation, it might be useful to comment briefly on the use of laboratory data, such as those presented in figure 7, in making quantitative predictions about the corresponding properties of clouds. The experiments of Rouse & Dodu indicated a dependence of mixing rate on both Reynolds number and a stability parameter. It seems reasonable to suppose, however, that the major effect of viscosity is to cause a decay in the intensity of turbulence between the stirrer and the interface. Thus if the properties of the turbulence are known very close to an interface, across which there is a given density difference, we should expect the rate of mixing and the structure of the mixing layer to depend mainly on the Richardson number, defined by



Linear Non-linear Cloud top

FIGURE 8. The relation between the region of stable density gradient and the 'visible edge' for the linear and non-linear laboratory experiments and the top of a stratocumulus cloud layer. Schematic density profiles have been superimposed on cross-sections of the interfaces.

$$Ri = \Delta\lambda/V_t^2, \tag{2}$$

where $\Delta = g(\rho_1-\rho_2)/\rho_2$, ρ_1 and ρ_2 are the densities of the bottom and top fluid respectively, and λ and V_t are length and velocity scales for the turbulence.

This idea is in accord with the results of Ellison & Turner (1959), who discussed the mixing produced by the flow of a dense turbulent layer under a deeper tank of fluid at rest. The method of production of the turbulence is different, but in both cases there is a sharp interface between turbulent and non-turbulent fluid. Their paper established a relation between the entrainment constant, or ratio of inflow velocity to mean velocity, and a Richardson number. In just the same way, we suggest that figure 7 could be made dimensionless and regarded as a plot of an entrainment constant against Richardson number, or

$$E = V/V_t = f_1(Ri), \tag{3}$$

where V is the velocity of the interface, and in our experiments λ and V_t are fixed. The presence of non-linear effects of course introduces another density ratio D, say the ratio of the maximum non-linearity to the density difference, but this has already been shown to have a small effect on E. An extension of the dimensional argument gives also an expression for the depth h of the stable transition region in the non-linear case

$$h/\lambda = f_2(Ri, D), \tag{4}$$

where f_2 is another (unknown) function of Ri and D.

Our laboratory data are clearly not extensive enough to warrant a detailed quantitative comparison with clouds, but the following order of magnitude estimates lead to plausible numbers. The density differences and degree of non-linearity are comparable in the two cases, so we need to compare only the relevant velocities and lengths. Equation (4) suggests that in the absence of information about the length scale λ of the turbulence, a Richardson number could be based instead on the length h, the depth of the stable layer. In the laboratory, typical values are $h = 1$ cm, $V_t = 1$ cm/sec, and in clouds, $h = 100$ m, $V_t = 1$ m/sec; and these sets of numbers in fact give the same value of the Richardson number for the same density difference. Thus applying our laboratory results to (3), we can say that in either case the velocity of the interface with 1 % density difference will be about $V_t/100$, or of the order 1 cm/sec for clouds, which is again a reasonable value.

Returning now to our observation of a reduction in mixing rate by the non-linear density changes, it may be seen that this throws some light on the nature of the mixing process. One might have expected from the observations without stirring that the existence of non-linear density changes producing heavier fluid could only increase the mixing rate, but instead there is a reduction. This must mean that the most important part of the mixing action takes place as an element of heavy fluid is thrown upwards into the lighter layer at rest. Such an element falls back into the stirred layer carrying some of the lighter fluid with it, which is then mixed quickly with the whole of the bottom layer. A non-linear density change makes the rising eddy heavier than it would otherwise be, so that it has a greater restoring force on it and falls back sooner (c.f. the observation of the smoother interface, without ragged eddies projecting a long way into the top fluid). The tendency to move faster downwards through its surroundings will of course persist in the lower layer, but it will be less important there where the mixture is more diluted and the intensity of turbulence higher.

We should emphasize again in conclusion that these results can only be applied to fluids, and in particular clouds, where mixing is mainly due to turbulence driven by processes independent of the evaporation itself. If the cooling due to evaporation contributes a substantial proportion of the energy of the turbulence then of course the mixing rate can be increased by the non-linear density change.

We are grateful to Mr L. F. Clague for his assistance with the experiments, and especially for designing our experimental tank.

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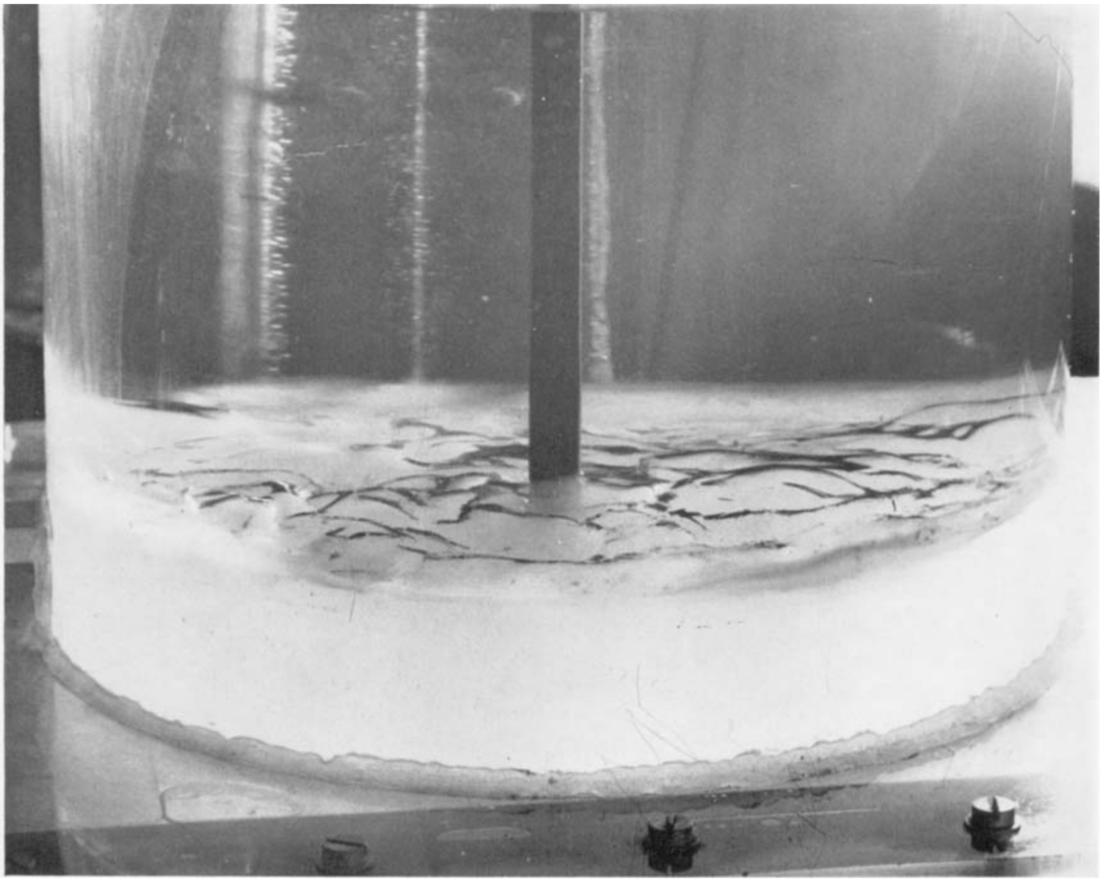

FIGURE 4. (PLATE 1). Photograph of the top of the interface during a non-linear mixing experiment. The density difference was about 3 %.